QUIZ 11 SOLUTIONS: LESSONS 15-16
OCTOBER 6, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] Determine whether

$$\int_0^1 \frac{1}{x^2}\,dx$$

converges or diverges. If it converges, find its value.

Solution: The function $\frac{1}{x^2}$ does not exist at $x = 0$, so this is the point we need to consider via a limit. Write

$$\int_0^1 \frac{1}{x^2}\,dx = \lim_{s\to 0^+}\int_s^1 \frac{1}{x^2}\,dx.$$

So

$$\begin{aligned}\int_s^1 \frac{1}{x^2}\,dx &= -\frac{1}{x}\Big|_s^1 \\ &= -\frac{1}{1} - \left(-\frac{1}{s}\right) \\ &= \frac{1}{s} - 1.\end{aligned}$$

Taking the limit,

$$\int_0^1 \frac{1}{x^2}\,dx = \lim_{s\to 0^+}\left(\frac{1}{s} - 1\right) = \infty - 1 = \infty.$$

Therefore, we conclude the integral $\boxed{\text{diverges}}$.

2. [2 pts] Find the 4^{th} partial sum of

$$\sum_{n=0}^{\infty} \frac{2}{n+2}.$$

Round your answer to 2 decimal places.

Solution: The 4^{th} partial sum is

$$\begin{aligned}
&\frac{2}{0+2}+\frac{2}{1+2}+\frac{2}{2+2}+\frac{2}{3+2}\\
&=\frac{2}{2}+\frac{2}{3}+\frac{2}{4}+\frac{2}{5}\\
&=1+\frac{2}{3}+\frac{1}{2}+\frac{2}{5}\\
&=\frac{3}{2}+\frac{10}{15}+\frac{6}{15}\\
&=\frac{3}{2}+\frac{16}{15}\\
&=\frac{45}{30}+\frac{32}{30}\\
&=\frac{77}{30}\approx\boxed{2.57}.
\end{aligned}$$

3. [4 pts] Determine if the following geometric series converges or diverges (give a reason for how you know it converges or diverges). If it converges, find its sum.

$$\sum_{n=1}^{\infty}\frac{3}{2}\left(-\frac{2}{3}\right)^n$$

Solution: Because $|r|=\left|-\frac{2}{3}\right|<1$, the series $\boxed{\text{converges}}$. We compute its sum. Write

$$\begin{aligned}
\sum_{n=1}^{\infty}\frac{3}{2}\left(-\frac{2}{3}\right)^n &= \sum_{n=0}^{\infty}\frac{3}{2}\left(-\frac{2}{3}\right)^{n+1}\\
&=\sum_{n=0}^{\infty}\frac{3}{2}\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)^n\\
&=\sum_{n=0}^{\infty}(-1)\left(-\frac{2}{3}\right)^n\\
&=\frac{-1}{1-\left(-\frac{2}{3}\right)}\\
&=\frac{-1}{1+\frac{2}{3}}\\
&=\frac{-3}{3+2}\\
&=\boxed{-\frac{3}{5}}
\end{aligned}$$